# Exercise 1.2 (Solutions)
# Mathematics 11 (PECTAA)

## Equality of two complex numbers

Let $x_1 = a + bi$ and $x_2 = c + di$. If $x_1 = x_2$, then

$$a + bi = c + di \iff a = c \text{ and } b = d.$$

Question 1: Find the value of $x$ and $y$ in each of the following:

$$\begin{aligned}
x + y + 2 - 3i^2 &= i(5 - i)(3 + 4i) \\
(x + 2) + i(y - 3) &= i(15 + 20i - 3i^2 - 4i^2) \\
(x + 2) + i(y - 3) &= i(19 + 17i) \\
(x + 2) + i(y - 3) &= -17 + 19i \\
x + 2 = -17 \quad &\text{and} \quad y - 3 = 19 \\
x = -17 - 2 \quad &\text{and} \quad y = 19 + 3 \\
x = -19 \quad &\text{and} \quad y = 22
\end{aligned}$$

**ii:**

$$(x + iy)(1 - i) = (2 - 3i)(-5 + 5i)\left(-i \cdot \frac{3}{5}\right)$$

$$\begin{aligned}
\text{Step 1: } (2 - 3i)(-5 + 5i) &= (2 \cdot -5 - (-3 \cdot 5)) + (2 \cdot 5 + (-3) \cdot -5)i \\
&= 5 + 25i
\end{aligned}$$

$$\begin{aligned}
\text{Step 2: Multiply by } -\frac{3}{5}i: \quad (5 + 25i)\left(-\frac{3}{5}i\right) &= -3i + 25 \cdot \frac{-3}{5}i^2 \\
&= -3i + 15 = 15 - 3i
\end{aligned}$$

$$\begin{aligned}
\text{Step 3: Solve for } x + iy: \quad x + iy &= \frac{15 - 3i}{1 - i} = \frac{(15 - 3i)(1 + i)}{(1 - i)(1 + i)} \\
&= \frac{18 + 12i}{2} = 9 + 6i
\end{aligned}$$

Final Answer: $x = 9,\ y = 6$

**iii:**

$$\frac{x}{2+i} + \frac{y}{3-i} = 4 + 5i$$

Step 1: Rationalize the denominators: $\frac{x}{2+i} = \frac{x(2-i)}{5}, \quad \frac{y}{3-i} = \frac{y(3+i)}{10}$

Step 2: Multiply through by 10: $2 \cdot x(2-i) + y(3+i) = 40 + 50i$

Step 3: Expand: $2x(2-i) + y(3+i) = (4x+3y) + (-2x+y)i = 40 + 50i$

Step 4: Equate real and imaginary parts: $4x + 3y = 40, \quad -2x + y = 50$

Step 5: Solve the system:
$y = 50 + 2x$
$4x + 3(50 + 2x) = 40$
$10x + 150 = 40$
$x = -11, \quad y = 50 + 2(-11) = 28$

Final Answer: $x = -11,\ y = 28$

## Question 2:Try yourself

## Square Root of Complex Number

If $z = x + yi$, then

$$\sqrt{z} = \pm\left(\sqrt{\frac{|z|+x}{2}} + i \cdot \frac{y}{|y|}\sqrt{\frac{|z|-x}{2}}\right),$$

where

$$|z| = \sqrt{x^2 + y^2}.$$

**Question 3: Find real $x$ and $y$ such that $(x + yi)^2 = 25 + 60i$**

$$x + yi = \pm\sqrt{25 + 60i}$$

Let $z = 25 + 60i$, then

$$|z| = \sqrt{25^2 + 60^2} = 65$$

$$\sqrt{25+60i} = \pm\left(\sqrt{\frac{65+25}{2}} + i \cdot \frac{60}{|60|}\sqrt{\frac{65-25}{2}}\right)$$

$$= \pm\left(\sqrt{45} + i\sqrt{20}\right) = \pm(3\sqrt{5} + 2\sqrt{5}i)$$

So,

$$x = \pm 3\sqrt{5}, \quad y = \pm 2\sqrt{5}.$$

**ii:** $(x+yi)^2 = 64+48i$

$$x+yi = \pm\sqrt{64+48i}$$

Let $z = 64+48i$, then

$$|z| = \sqrt{64^2+48^2} = 80$$

$$\sqrt{64+48i} = \pm\left(\sqrt{\frac{80+64}{2}} + i\cdot\frac{48}{|48|}\sqrt{\frac{80-64}{2}}\right)$$

$$= \pm\left(\sqrt{72}+i\sqrt{8}\right) = \pm(6\sqrt{2}+2\sqrt{2}i)$$

So,

$$x = \pm 6\sqrt{2}, \quad y = \pm 2\sqrt{2}.$$

part (iii) Try yourself

## Question 4

$$z_1 = 2+3i, \qquad z_2 = 1$$

We want: $\mathrm{Im}(z_1 z_2) = 7$

Step 1: Multiply $z_1$ and $z_2$ : $z_1 z_2 = (2+3i)(1-\alpha) = 2(1-\alpha)+3i(1-\alpha)$

$$= 2-2\alpha+(3-3\alpha)i$$

Step 2: Extract the imaginary part: $\mathrm{Im}(z_1 z_2) = 3-3\alpha$

Step 3: Set imaginary part equal to 7 and solve for $\alpha$ : $3-3\alpha = 7$

$$-3\alpha = 4$$

$$\alpha = -\frac{4}{3}$$

Final Answer: $\alpha = -\frac{4}{3}$

## Question 5: Try yourself

## Question 6: Show that

## Proof: $\forall z_1, z_2 \in \mathbb{C},\ \overline{z_1 z_2} = \overline{z_1}\cdot\overline{z_2}$

Let $z_1 = a+bi$, $z_2 = c+di$. Then

$$z_1 z_2 = (a+bi)(c+di) = ac-bd+i(bc+ad)$$

$$\overline{z_1 z_2} = ac-bd-i(bc+ad)$$

$$\overline{z_1}\cdot\overline{z_2} = (a-bi)(c-di) = ac-adi-bci+bdi^2 = ac-bd-i(ad+bc)$$

Hence,

$$\overline{z_1 z_2} = \overline{z_1}\cdot\overline{z_2}.$$

## Question 7: Find the Square Roots of the following Complex Numbers

**(1)** $-7-24i$

Let $z=-7-24i$, $|z|=25$

$$\sqrt{z}=\pm\left(\sqrt{\frac{25+(-7)}{2}}+i\cdot\frac{-24}{|-24|}\sqrt{\frac{25-(-7)}{2}}\right)$$

$$=\pm\left(\sqrt{9}-i\sqrt{16}\right)=\pm(3-4i)$$

**(2)** $8-6i$

Let $z=8-6i$, $|z|=10$

$$\sqrt{z}=\pm\left(\sqrt{\frac{10+8}{2}}+i\cdot\frac{-6}{|-6|}\sqrt{\frac{10-8}{2}}\right)$$

$$=\pm(3-i)$$

**(3)** $-15-36i$

Let $z=-15-36i$, $|z|=39$

$$\sqrt{z}=\pm\left(\sqrt{\frac{39+(-15)}{2}}-i\sqrt{\frac{39-(-15)}{2}}\right)$$

$$=\pm\left(\sqrt{12}-i\sqrt{27}\right)=\pm(2\sqrt{3}-3i\sqrt{3})$$

**(4)** $119+120i$

Let $z=119+120i$, $|z|=169$

$$\sqrt{z}=\pm\left(\sqrt{\frac{169+119}{2}}+i\sqrt{\frac{169-119}{2}}\right)$$

$$=\pm(12+5i)$$

## Question 9:Find real $x$ and $y$ from equation:

$$(-7+i)(x+iy)+(-1-5i)=i(11-i)$$

Step 1: Expand $(-7+i)(x+iy)$: $(-7+i)(x+iy)=-7x-7iy+ix+i^2y=(-7x-y)+(-7y+x)i$

Add $(-1-5i)$: $(-7x-y-1)+(-7y+x-5)i$

Step 2: Simplify the RHS: $i(11-i)=11i-i^2=1+11i$

Step 3: Equate real and imaginary parts:
$$-7x-y-1=1 \quad \Rightarrow \quad -7x-y=2$$
$$-7y+x-5=11 \quad \Rightarrow \quad x-7y=16$$

Step 4: Solve the system:
$$x=16+7y$$
$$-7(16+7y)-y=2 \quad \Rightarrow \quad -112-49y-y=2 \quad \Rightarrow \quad -50y=114$$
$$y=-\frac{57}{25}, \quad x=16+7(-\frac{57}{25})=\frac{1}{25}$$

Final Answer: $x=\frac{1}{25},\ y=-\frac{57}{25}$

## Question 10:Find real $x$ and $y$ from equation:

if (5-2i)(x+iy)+3 =i(11-i)-4i

$$(5-2i)(x+iy)+3=i(11-i)-4i$$

Step 1: Expand the LHS:
$$(5-2i)(x+iy)=5x+5iy-2ix-2i^2y=(5x+2y)+(5y-2x)i$$
$$(5-2i)(x+iy)+3=(5x+2y+3)+(5y-2x)i$$

Step 2: Simplify the RHS: $i(11-i)-4i=11i-i^2-4i=1+7i$

Step 3: Equate real and imaginary parts:
$$5x+2y+3=1 \quad \Rightarrow \quad 5x+2y=-2$$
$$5y-2x=7$$

Step 4: Solve the system:
$$x=\frac{5y-7}{2}$$
$$5\left(\frac{5y-7}{2}\right)+2y=-2 \quad \Rightarrow \quad \frac{25y-35+4y}{2}=-2$$
$$29y-35=-4 \quad \Rightarrow \quad y=\frac{31}{29}$$
$$x=\frac{5\cdot\frac{31}{29}-7}{2}=-\frac{24}{29}$$

Final Answer: $x=-\frac{24}{29},\ y=\frac{31}{29}$

**Question 11:Find real $x$ and $y$ from equation: same Question no 10 Try yourself**

**Question 12: same Question no 4 Try yourself**