Exercise 1.3 (Solutions)
Mathematics 11

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Question 1: Factorize the following

(i)

$$a^2 + 4b^2 = a^2 + (2b)^2$$
$$= (a + 2bi)(a - 2bi).$$

$$\text{If } a^2 + 4b^2 = 0, \text{ then } a^2 = -4b^2.$$
$$\implies a = 0,\ b = 0 \quad \text{(real solution).}$$
$$\text{Or } a = \pm 2bi \quad \text{(complex solution).}$$

(ii)

$$3x^2 + 3y^2 = 3(x^2 + y^2)$$
$$= 3(x + iy)(x - iy).$$

$$\text{If } 3x^2 + 3y^2 = 0, \text{ then } x^2 + y^2 = 0.$$
$$\implies x = 0,\ y = 0 \quad \text{(real solution),}$$
$$\text{or } x = \pm iy \quad \text{(complex solution).}$$

(iii)

$$
\begin{aligned}
9a^2 + 16b^2 &= (3a)^2 + (4b)^2 \\
&= (3a + 4bi)(3a - 4bi).
\end{aligned}
$$

$$
\begin{aligned}
\text{If } 9a^2 + 16b^2 &= 0, \\
&\implies (3a)^2 = -(4b)^2, \\
&\implies a = \pm\tfrac{4i}{3}b.
\end{aligned}
$$

$$
\begin{aligned}
\text{Hence, } a = 0,\ b = 0 \text{ (real solution)}, \\
\text{or } a = \pm\tfrac{4i}{3}b \text{ (complex solution)}.
\end{aligned}
$$

(iv)

$$
\begin{aligned}
144x^2 + 225y^2 &= (12x)^2 + (15y)^2 \\
&= (12x + 15iy)(12x - 15iy).
\end{aligned}
$$

$$
\begin{aligned}
\text{If } 144x^2 + 225y^2 &= 0, \\
&\implies (12x)^2 = -(15y)^2, \\
&\implies x = \pm\tfrac{5i}{4}y.
\end{aligned}
$$

$$
\begin{aligned}
\text{Hence, } x = 0,\ y = 0 \text{ (real solution)}, \\
\text{or } x = \pm\tfrac{5i}{4}y \text{ (complex solution)}.
\end{aligned}
$$

(iv)

$$\begin{aligned} z^2 - 2zi - 1 &= 0 \\ z &= \frac{-(-2i) \pm \sqrt{(-2i)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{2i \pm \sqrt{-4+4}}{2} \\ &= \frac{2i}{2} = i. \end{aligned}$$

$$\text{Hence, the repeated root is } z = i, \quad z^2 - 2zi - 1 = (z-i)^2.$$

Remaining parts Try yourself

Question 2: Factorize the following polynomial into its linear factors:

(i)

$z^3 + 8$
$z^3 + 2^3$
$(z+2)(z^2 - 2z + 4)$
$(z+2)(z^2 - 2z + 1 + 3)$
$(z+2)\left((z-1)^2 - (\sqrt{3}i)^2\right)$
$(z+2)(z - 1 - \sqrt{3}i)(z - 1 + \sqrt{3}i)$

(ii)

$z^3 + 27$
$z^3 + 3^3$
$(z+3)(z^2 - 3z + 9)$
$(z+3)\left(z^2 - 3z + \left(\frac{3}{2}\right)^2 + 9 - \left(\frac{3}{2}\right)^2\right)$
$(z+3)\left(\left(z - \frac{3}{2}\right)^2 + \frac{27}{4}\right)$
$(z+3)\left(\left(z - \frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}i\right)^2\right)$
$(z+3)\left(z - \frac{3}{2} - \frac{3\sqrt{3}}{2}i\right)\left(z - \frac{3}{2} + \frac{3\sqrt{3}}{2}i\right)$

(iii)

$z^3 - 2z^2 + 16z - 32$
$z^2(z-2) + 16(z-2)$
$(z-2)(z^2+16)$
$(z-2)(z^2-(4i)^2)$
$(z-2)(z-4i)(z+4i)$

(iv)

$z^4 + 21z^2 - 100$
$z^4 + 25z^2 - 4z^2 - 100$
$z^2(z^2+25) - 4(z^2+25)$
$(z^2+25)(z^2-4)$
$(z^2-(5i)^2)(z^2-2^2)$
$(z-5i)(z+5i)(z-2)(z+2)$

(v)

$z^4 - 16$
$(z^2)^2 - 4^2$
$(z^2-4)(z^2+4)$
$(z^2-2^2)(z^2-(2i)^2)$
$(z-2)(z+2)(z-2i)(z+2i)$

(vi)

$z^4 + 3z^2 - 4$
$z^4 + 4z^2 - z^2 - 4$
$z^2(z^2+4) - 1(z^2+4)$
$(z^2+4)(z^2-1)$
$(z^2-(2i)^2)(z^2-1^2)$
$(z-2i)(z+2i)(z-1)(z+1)$

(vii)

$z^4 + 5z^2 + 6$
$z^4 + 3z^2 + 2z^2 + 6$

$z^2(z^2+3)+2(z^2+3)$
$(z^2+3)(z^2+2)$
$(z^2-(\sqrt{3}i)^2)(z^2-(\sqrt{2}i)^2)$
$(z-\sqrt{3}i)(z+\sqrt{3}i)(z-\sqrt{2}i)(z+\sqrt{2}i)$

(viii)

z^4-32z^2-3469
$z^4-81z^2+49z^2-3969$
$z^2(z^2-81)+49(z^2-81)$
$(z^2+49)(z^2-81)$
$(z^2-(7i)^2)(z^2-9^2)$
$(z-7i)(z+7i)(z-9)(z+9)$

Question 3: Find the roots of $z^4+7z^2-144=0$ and hence factorize into product of linear factors:

$$z^4+7z^2-144=0$$
$$z^4+16z^2-9z^2-144=0$$
$$z^2(z^2+16)-9(z^2+16)=0$$
$$(z^2-9)(z^2+16)=0$$
$$(z-3)(z+3)(z-4i)(z+4i)=0$$

Question 4: Solve the following complex quadratic equation by completing square method:

(i)

$z^2-3z+4=0$
$z^2-3z=-4$
$z^2-2\cdot\frac{3}{2}z+\left(\frac{3}{2}\right)^2=-4+\left(\frac{3}{2}\right)^2$
$\left(z-\frac{3}{2}\right)^2=-\frac{7}{4}$
$z-\frac{3}{2}=\pm\frac{\sqrt{7}}{2}i$
$z=\frac{3\pm\sqrt{7}i}{2}$
Solution set: $\left\{\frac{3+\sqrt{7}i}{2},\frac{3-\sqrt{7}i}{2}\right\}$

(ii)

$z^2 - 6z + 30 = 0$
$z^2 - 6z = -30$
$z^2 - 2 \cdot 3z + 3^2 = -30 + 9$
$(z-3)^2 = -21$
$z - 3 = \pm\sqrt{21}i$
$z = 3 \pm \sqrt{21}i$
Solution set: $\left\{3 + \sqrt{21}i, 3 - \sqrt{21}i\right\}$

(iii)

$3z^2 - 18z + 50 = 0$
$z^2 - 6z + \frac{50}{3} = 0$
$z^2 - 6z = -\frac{50}{3}$
$z^2 - 2 \cdot 3z + 3^2 = -\frac{50}{3} + 9$
$(z-3)^2 = -\frac{23}{3}$
$z - 3 = \pm\sqrt{\frac{23}{3}}i$
$z = 3 \pm \frac{\sqrt{69}}{3}i$

(iv)

$z^2 + 4z + 13 = 0$
$z^2 + 4z = -13$
$z^2 + 2 \cdot 2z + 2^2 = -13 + 4$
$(z+2)^2 = -9$
$z + 2 = \pm 3i$
$z = -2 \pm 3i$
Solution set: $\{-2 + 3i, -2 - 3i\}$

(v)

$2z^2 + 6z + 9 = 0$
$z^2 + 3z + \frac{9}{2} = 0$
$z^2 + 3z = -\frac{9}{2}$
$z^2 + 2 \cdot \frac{3}{2}z + \left(\frac{3}{2}\right)^2 = -\frac{9}{2} + \frac{9}{4}$
$\left(z + \frac{3}{2}\right)^2 = -\frac{9}{4}$
$z + \frac{3}{2} = \pm\frac{3}{2}i$

$z = -\frac{3}{2} \pm \frac{3}{2}i$
Solution set: $\left\{\frac{-3+3i}{2}, \frac{-3-3i}{2}\right\}$

(vi)

$3z^2 - 5z + 7 = 0$
$z^2 - \frac{5}{3}z + \frac{7}{3} = 0$
$z^2 - \frac{5}{3}z = -\frac{7}{3}$
$z^2 - 2 \cdot \frac{5}{6}z + \left(\frac{5}{6}\right)^2 = -\frac{7}{3} + \frac{25}{36}$
$\left(z - \frac{5}{6}\right)^2 = -\frac{59}{36}$
$z - \frac{5}{6} = \pm\frac{\sqrt{59}}{6}i$
$z = \frac{5 \pm \sqrt{59}i}{6}$

Question 5: Solve the following equations:

(i)

$2z^4 - 32 = 0$
$2(z^4 - 16) = 0$
$z^4 - 16 = 0$
$(z^2 - 4)(z^2 + 4) = 0$
$(z - 2)(z + 2)(z - 2i)(z + 2i) = 0$

(ii)

$3z^5 - 243z = 0$
$3z(z^4 - 81) = 0$
$3z(z^2 - 9)(z^2 + 9) = 0$
$3z(z - 3)(z + 3)(z - 3i)(z + 3i) = 0$
Roots: $0, 3, -3, 3i, -3i$

(iii)

$z^5 - z = 0$
$z(z^4 - 1) = 0$
$z(z^2 - 1)(z^2 + 1) = 0$
$z(z - 1)(z + 1)(z - i)(z + i) = 0$
Roots: $0, 1, -1, i, -i$

(iv)

$z^3 - 5z^2 + z - 5 = 0$
$z^2(z-5) + 1(z-5) = 0$
$(z-5)(z^2+1) = 0$
$(z-5)(z-i)(z+i) = 0$
Roots: $5, i, -i$

(v)

$4z^4 - 25z^2 - 39 = 0$
$4z^4 - 28z^2 + 3z^2 - 21 = 0$
$4z^2(z^2-7) + 3(z^2-7) = 0$
$(z^2-7)(4z^2+3) = 0$
$(z-\sqrt{7})(z+\sqrt{7})(2z-\sqrt{3}i)(2z+\sqrt{3}i) = 0$

(vi)

$z^3 + z^2 + z + 1 = 0$
$z^2(z+1) + 1(z+1) = 0$
$(z+1)(z^2+1) = 0$
$(z+1)(z-i)(z+i) = 0$
Roots: $-1, i, -i$

Q6. Find the polynomial $P(z)$ of degree 3 with zeros $3, 2i, -2i$ and satisfying $P(1) = 20$.

Let $P(z) = a(z-3)(z-2i)(z+2i)$
$P(z) = a(z-3)(z^2+4)$
$P(1) = a(1-3)(1+4) = -10a$
Given $P(1) = 20 \Rightarrow -10a = 20 \Rightarrow a = -2$
So, $P(z) = -2(z-3)(z^2+4) = -2z^3 + 6z^2 - 8z + 24$

Q7. Find a polynomial $P(z)$ of degree 4 with zeros $2, -2, i, -i$ and satisfying $P(3) = 240$.

Let $P(z) = a(z-2)(z+2)(z-i)(z+i)$
$P(z) = a(z^2-4)(z^2+1)$
$P(3) = a(9-4)(9+1) = a \cdot 5 \cdot 10 = 50a$

Given $P(3) = 240 \Rightarrow 50a = 240 \Rightarrow a = \frac{24}{5}$
So, $P(z) = \frac{24}{5}(z^2 - 4)(z^2 + 1) = \frac{24}{5}(z^4 - 3z^2 - 4)$

Q8. Find the polynomial of degree 4 with zeros $4, -4, 1+2i, 1-2i$ and satisfying $P(2) = 72$.

Zeros: $4,\ -4,\ 1+i,\ 1-i.$

$P(x) = k(x-4)(x+4)(x-(1+i))(x-(1-i))$

$(x-(1+i))(x-(1-i)) = (x-1)^2 + 1 = x^2 - 2x + 2, \qquad (x-4)(x+4) = x^2 - 16$

$P(x) = k(x^2 - 16)(x^2 - 2x + 2).$

$P(2) = k(4-16)(4-4+2) = k(-12)(2) = -24k = 72 \implies k = -3.$

$$\boxed{P(x) = -3(x^2 - 16)(x^2 - 2x + 2) = -3x^4 + 6x^3 + 42x^2 - 96x + 96}$$