

## How to Solve Quadratic Equations (Step by Step)

A Complete Guide for Students (Grade 9–12)

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### Introduction

Quadratic equations are everywhere — from the way a ball moves in the air to how businesses calculate profit.

That's why they're one of the most important topics in algebra.

This guide is designed to walk you through quadratic equations step by step.

By the time you finish, you won't just know how to solve them, you'll actually understand why each method works.

We'll explore factorization, completing the square, the quadratic formula, and even graphical methods — all explained in simple language with examples.

### The Standard Form of a Quadratic Equation

A quadratic equation is simply a polynomial of degree 2. Its general form looks like this:

$$ax^2 + bx + c = 0, \text{ where } a \neq 0$$

Example:  $2x^2 + 3x - 5 = 0$

#### Method 1: Factorization

Steps:

1. Multiply a and c
2. Find two numbers whose product = ac and sum = b
3. Break the middle term
4. Factorize by grouping

Example:  $x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0 \rightarrow x = -2, -3$

#### Method 2: Completing the Square

Steps:

1. Start with  $ax^2 + bx + c = 0$
2. Divide through by a (if  $a \neq 1$ )
3. Isolate constant term
4. Add and subtract  $(b/2a)^2$
5. Solve for x

Example:  $x^2 + 6x + 5 = 0 \rightarrow x = -1, -5$

### Method 3: Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:  $2x^2 + 3x - 5 = 0 \rightarrow x = 1, -2.5$

### Method 4: Graphical Method

Graph  $y = ax^2 + bx + c$  (a parabola). The x-intercepts are the solutions.

### Applications of Quadratic Equations

- Projectile Motion
- Business (profit & cost)
- Geometry (areas & dimensions)

### Practice Problems

1.  $x^2 + 7x + 12 = 0$
2.  $2x^2 - 5x - 3 = 0$
3.  $x^2 - 2x - 15 = 0$
4.  $3x^2 + 2x - 1 = 0$
5.  $x^2 + 4x + 1 = 0$  (use completing the square)

### Quick Formula Sheet

- Standard Form:  $ax^2 + bx + c = 0$
- Factorization: split middle term
- Completing the Square: add  $(b/2a)^2$
- Quadratic Formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Nature of Roots:

$D > 0 \rightarrow$  Two real distinct roots

$D = 0 \rightarrow$  Two equal roots

$D < 0 \rightarrow$  No real roots (complex)