System is called curvelinear coordinate system.

=) If u_1, u_2, u_3 are orthogonal curvelinear coordinate then ① $|\nabla u_j| = h_j^{-1}$

② $\hat{e}_j = \hat{E}_j$, $J = 1, 2, 3$

curvelinear coordinate system

Arc length

$$\vec{r} = r(u_1, u_2, u_3)$$

$$dr = \frac{\partial r}{\partial u_1} du_1 + \frac{\partial r}{\partial u_2} du_2 + \frac{\partial r}{du_3} du_3$$

$$= h_1 e_1 du_1 + h_2 e_2 du_2 + e_3 h_3 du_3$$

arc length ds form

$$(ds)^2 = dr \cdot dr = h_1^2 (du_1)^2 + h_2^2 (du_2)^2 + h_3^2 (du_3)^2$$

$\because \hat{e_1} \cdot \hat{e_1} = \hat{e_2} \cdot \hat{e_2} = \hat{e_3} \cdot \hat{e_3} = 1$

Arc element

$$dA_1 = |h_2 du_2 \hat{e_2}) \times (h_3 du_3 \hat{e_3}|$$

$\because |\hat{e_2} \times \hat{e_3}| = |\hat{e_1}| = 1$

$$= h_2 h_3 |\hat{e_2} \times \hat{e_3}| du_2 du_3 = h_2 h_3 du_2 du_3$$

$$dA_2 = |(h_1 \hat{e_1} du_1) \times h_3 \hat{e_3} du_3|$$

$$= h_1 h_3 du_1 du_3$$

$$dA_3 = |h_1 du_1 \hat{e_1} \times h_2 du_2 e_2|$$

$$= h_1 h_2 du_1 du_2$$

Volume element

$$dv = |h_1 \hat{e_1} du_1 \cdot \hat{e_2} h_2 du_2 \times (\hat{e_3} h_3 du_3)|.$$

$$= |h_1 h_2 h_3 du_1 du_2 du_3 \; e_1 \cdot e_2 \times e_3|$$

Jacobian in orthogonal $\quad \because |e_1 \cdot e_2 \times e_3| = 1$

$$\frac{\partial(x, y, z)}{\partial(u_1, u_2, u_3)} = \begin{vmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial y}{\partial u_1} & \frac{\partial z}{\partial u_1} \\ \frac{\partial x}{\partial u_2} & \frac{\partial y}{\partial u_2} & \frac{\partial z}{\partial u_2} \\ \frac{\partial x}{\partial u_3} & \partial y/\partial u_3 & \frac{\partial z}{\partial u_3} \end{vmatrix}$$