v.v.v.

If $\nabla \times A = 0$ in Some Region R then $\vec{A}$ is called irrotational

Theorem

(i) $\nabla \times (A+B) = \nabla \times A + \nabla \times B$

(ii) $\nabla \times (\varphi A) = \varphi(\nabla \times A) + (\nabla \varphi) A$

(iii) $\nabla \times (\nabla \varphi) = 0$ (curl grad $\varphi = 0$).

$\nabla \cdot (\nabla \times A) = 0$ (div curl $A \neq 0$)

Q: divergence

div $F = \nabla \cdot F$ $\therefore \nabla \cdot A \neq A \cdot \nabla$

v.v.v.Q curl

$\nabla \times F$

irrotational vector or Conservative force

$\nabla \times F \neq 0$ or curl $\times F = 0$

vector identity

$\Rightarrow$ $\operatorname{div}(\varphi \bar{A}) = \operatorname{grad}\varphi \cdot A - \varphi \operatorname{div}(A)$

$\operatorname{div}(A \times B) = B \cdot (\operatorname{curl} A) - A \cdot (\operatorname{curl} B)$ $\therefore \operatorname{grad}\varphi = \nabla \varphi$

$\operatorname{curl}(\operatorname{grad}\varphi) = 0$ $\nabla \varphi = \frac{\partial \varphi}{\partial x} i + \frac{\partial \varphi}{\partial y} j + \frac{\partial \varphi}{\partial z} k$

or

$\Rightarrow \nabla \cdot \nabla \varphi = 0$ $\nabla \cdot A = \operatorname{div} A$

$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$

$\Rightarrow$ $\operatorname{div}(\operatorname{curl} A) = 0$

$\nabla \cdot \nabla \times A = 0$

$\Rightarrow$ $\operatorname{curl} A = \operatorname{grad}(\operatorname{div} A) - \nabla^2 A$

$\nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$

$\Rightarrow$ $\operatorname{curl}(\operatorname{curl} A) - \operatorname{grad}(\operatorname{div} A) - \nabla^2 A$

$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$\nabla^2 \varphi = 0$ is Laplace