irrotational

v.v. imp

If $\nabla \cdot A = 0$ everywhere in the region R then A is called solenoidal vector

Property of divergence

(i) $\nabla \cdot (A+B) = \nabla \cdot A + \nabla \cdot B$

(ii) $\nabla \cdot (\phi A) = \phi(\nabla \cdot A) + A(\nabla \cdot \phi)$

If ϕ is constant then $\nabla \cdot (\phi A) = \phi \nabla \cdot A$

(i) $\nabla \cdot r = 3$ $\Rightarrow r = xi + yj + zk$

$\nabla \cdot r = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z}(z)$

$1+1+1 = 3$

(ii) $\nabla \cdot [f(r)\bar{r}] = 3f(r) + r f'(r)$

(iii) $\nabla \cdot (r^n \vec{r}) = (n+3) r^n$

(iv) $\nabla \cdot \left(\frac{r}{r^3}\right) = 0$

Laplace

$\nabla^2 \phi = 0$

If $\nabla^2 \phi = 0$ in a certain Region then ϕ is said to be harmonic function

Theorem

(i) $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$

(ii) $\nabla^2 \left(\frac{1}{r}\right) = 0$ (iii) $\nabla^2 r^n = n(n+1) r^{n-2}$

Curl of a vector point function

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$