⇒ Infinite dim Subspace need not be closed e.g $X = [0,1]$

dim $X = n$

The Every finite dim Subspace Y of a normed Space X is closed in X.

Closed = convergent

$Y \subseteq X$ dim n (finite)

Y is complete

$x_n \to x$ convergent = cauchy

جس Space کے point vala convergent ہو تو closed ہوتی ہے

Def

$\|\cdot\| = \|\cdot\|_0$ is equivalent

$\|x\| = \|x\|_0$ (reflexive, symmetric & transitive)

$a\|x\|_0 \leq \|x\| \leq b\|x\|_0$

The on a finite dim vector Space X, any norm $\|\cdot\|$ is equivalent to any other norm $\|\cdot\|_0$

Proof dim $X = n$ $= \{e_1, e_2, \dots e_n\}$

$x = e_1\alpha_1 + e_2\alpha_2 \dots e_n\alpha_n$ (L.C.)

$\|\cdot\| \sim \|\cdot\|_0$

$$\|x\| = \left\|\sum_{i=1}^{n} \alpha_i e_i\right\|$$

$$\geq c \sum_{i=1}^{n} |\alpha_i| \quad — (1)$$

$$\|x\| = \left\|\sum_{i=1}^{n} \alpha_i e_i\right\| \leq \sum_{\alpha=1}^{n} |\alpha_i| \|e_i\| \leq K \sum_{i=1}^{n} |\alpha_i| \quad — (2)$$

$\therefore K = \max \|e_i\|_0$

$$\frac{\|x\|_0}{K} \leq \sum_{i=1}^{n} |\alpha_i| \leq \frac{\|x\|}{c}$$