complete ... space

Finite dim

$\dim X = n$ $\quad B = \{x_1, x_2 \ldots x_n\}$

$x \in X$, $x = \alpha_1 x_1 + \alpha_2 x_2 \ldots \alpha_n x_n$

<u>lemma</u>

$\{x_1, x_2 \ldots x_n\}$ l.g

$\exists\, c > 0$ For every $\alpha_i > 0$

$$\left\| \sum_{i=1}^{n} \alpha_i x_i \right\| \geq c \sum_{i=1}^{n} |\alpha_i| \quad \text{--- (1)}$$

$$\left\| \sum_{i=1}^{n} \alpha_i x_i \right\| \leq \sum_{i=1}^{n} \| \alpha_i x_i \| = \sum_{i=1}^{n} |\alpha_i| \, \|x_i\|$$

Thore

⇒ Every finite dim subspace Y of a normed space X is complete ⇒ (cauchy + converg)

keypt

⇒ (normed space may or may not complete

completeness = cauchy seq + converges)

⇒ $R^1, C^1, R^2, R^3, R^n, C^n$, $P[a,b]$ is complete ($n+1$)

all are finite dim (complete)

V.V.) The ⇒ If X - Infinite and $Y \subset X$ finite then X is finite therefor X is complete

The ⇒ Every finite dim normed space is complete

<u>Proof</u> X ⇒ normed $\quad Y \subseteq X$

$\dim Y = n$ finite

$\{e_1, e_2 \ldots e_n\}$ basis