... bounded operator

V.V.V A linear function f with $D(f)$ in a normed space X is continuous iff f is closed.

Functional

A functional is an operator whose range lies on the real line R or in C

Linear function

Let X be normed space over the field $K = R$ or C

If $f: D(f) \to K$ where $D(f)$ is a subspace of X is said linear if for all $x, y \in X$ and scalar α we have

(i) $f(x+y) = f(x) + f(y)$ (ii) $f(\alpha x) = \alpha f(x)$

Bounded linear function

A $f: D(f) \to K$ where $D(f)$ is a subset of normed space X is said to be a bounded L. f

(a) f is linear

(b) $|f(x)| \leq c\|x\|$ $\quad x \in D(f)$

$$\frac{|f(x)|}{\|x\|} ; \quad x \in D(f) - \{0\}$$

$$\|f\| = \sup_{x \in D(f)} \frac{|f(x)|}{\|x\|}$$

$$|f(x)| \leq \|f\| \|x\|$$

next pages

Metric Intro

When we say that

$x_n \to x$ in Real line

$-R$ ——— x ——— y ——— $+R$

i) $d(x,y) = |x-y|$ ii) $\sqrt{|x-y|}$ all in R

full calculus depend on distance

e.g

$d(R,R) = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

For all $x = R$; $x = R^2$, $x = R^2$

use distance

But

$X =$ Set of all infinite seq

e.g $x_n = \frac{1}{n}$, $y_n = 2^n$

$x_n = (-1)^n$

$X =$ Set of all matrices

If $X \neq \phi$

$d : X \times X \to [0, \infty) \Rightarrow$ non-negative

why we study metric space

used all above

Exp

(4) $d(x,y) = \frac{|x-y|}{1+|x-y|}$ is metric on R

(5) $d(x,y) = (|x-y|)^2$ not metric on R

Real line R

$d(x,y) = |x-y|$ is metric

$d(x,y) = \sqrt{|x_1 - y_1|^2 + |x_2 - y_2|^2}$

$d_1(x,y) = |x_1 - y_1| + |x_2 - y_2|$

$d_\infty(x,y) = \max(|x_1 - y_1|, |x_2 - y_2|)$

$d_p = \left(\sum_{i=1}^{2} |x_i - y_i|^p\right)^{1/p}$

$R, R^2, R^3 \ldots R^n$ are all metric and C

$z_1\ z_2 \in C$

$d: C \times C \to [0, \infty)$

$d(z_1, z_2) = |z_1 - z_2|$

C^n pe b metric defien

when we increase $R^n \to R^\infty$

$R^\infty = \{$ Set of all infinite sea of Real no $\}$

e.g

$d(x,y) = \left(\sum_{i=1}^{\infty} |x_i - y_i|^p\right)^{1/p}$

$= \sup_{i=1}^{\infty} |x_i - y_i|$

increasing function

$x_1 \le x_2 \Rightarrow f(x_1) \le f(x_2)$

distance

$P(R)$ = power set

$[0,1] \neq [1,2]$ — 0 1 2

$d(a,b) = d(1,1) = 0$

This exp is not distance

Product of metric

$X = X_1 \times X_2$ of two metric

$(X_1, d_1)(X_2, d_2)$

$$d(x,y) = d_1(x_1, y_1) + d_2(x_2, y_2)$$

Space l^∞

e.g. $x_i = \left(\frac{1}{n}\right) \in l^\infty$ $\quad y_i (-1)^n \in l^2$

$(-1, 1, -1, \ldots)$

$y \quad (1, \frac{1}{2}$

$$d(x,y) = \sup_{i=1}^{\infty} |x_i - y_i| \Rightarrow \sup \left| \frac{1}{n} - (-1)^n \right|$$

$$\sup\left(2, \frac{1}{2} \ldots\right)$$

Function space

$X = F[a\ b]$

Set of all function

we take continuous function

$$\max_{t \in [a,b]} |x(t) - y(t)|$$

Q Duina Ka Koi be non-empty set lea Jay Kia vo metric bun saata hai

ans Yes

e.g $X \neq \phi$ $X = \{a, b, c, d\}$

discrete metric is defined on every set

what is relation b/w discret metric and discrete topology

P If Series Converges then we defined metric

ℓ^p Space

$$\sum_{j=1}^{\infty} |x_i|^p < \infty$$

If $p = 1$

$\ell_1 = \sum_{j=1}^{\infty} |x_i|$ is convergent

$p = 2$

$\ell_2 = \sum_{i=1}^{\infty} |x_i|^2$ is convergent

ℓ^p is If convergent then we defined metric

e.g $\sum \frac{1}{n}$ divergent $= \ell_1$ but not convergent

$\sum \frac{1}{n^2}$ is convergent $x \in \ell^2$

=> If a one point then metric is 0

=> $Kd(x, y) = Kd$ iff $K > 0$

=> $d + K =$ not possible for nonzero real no

$\Rightarrow$ $A \subset l^{\infty}$ $x \in A$, $x = (0,1,0,1,0 \ldots)$

$x, y \in A$ $d(x,y) = 0$

then the induce metric is also metric on l^{∞}

$\Rightarrow$ metric $\tilde{d}$ on $C[a,b]$ is defined by

$$\tilde{d}(x,y) = \int_a^b |x(t) - y(t)| dt$$ is

also metric

$\Rightarrow$ discrete metric is also metric

Hamming distance

let X be set of all ordered triples of zero and 1

$(0,0,0)$ $(0,0,1)$ $(0,1,0)$ $(1,0,0)$ $(1,1,1)$ $(1,1,0)$ $(1,0,1)$ $(0,1,1)$ is also metric distance.

Triangle inequality

$$d(x_1, x_2) \leq d(x_1, x_2) + d(x_2, x_3) \ldots d(x_{n-1}, x_n)$$

is $K \geq 3$ is metric

$\Rightarrow$ $\delta(A) = 0$ iff A consist of a single point

$\Rightarrow$ if $A \cap B \neq 0$ then $D(A,B) = 0$ converse not hold

$\Rightarrow$ if $D(A,B) = 0$ then $A \cap B \neq 0$ not equal

$\Rightarrow$