(α = scalar (R, C))

let $y_n \to$ cauchy (R, C = complete)

$y_1, y_2, y_3 - \cdots y_m, y_n$

$y_1 = \alpha_1^{(1)} e_1 + \alpha_2^{(1)} e_2 + \cdots \alpha_n^{(1)} e_n$

$y_2 = \alpha_1^{(2)} e_1 + \alpha_2^{(2)} e_2 - \cdots \alpha_n^{(2)} e_n$

$\vdots$

$y_n = \alpha_1^{(1)} e_1 + \alpha_2^{(2)} e_2 - \cdots \alpha_n^{(3)} e_n$

Now we use cauchy seq

$\forall \varepsilon > 0 \ \exists \ N$ s.t

$d(y_m, y_r) < \varepsilon$

$\| y_m - y_r \| < \varepsilon$

$\varepsilon > \| y_m - y_r \|$

$= \| \sum_{i=1}^{n} \alpha_i^{m} e_i - \sum_{i=1}^{n} \alpha_i^{(r)} e_i \|$

$= \| \sum_{i=1}^{n} (\alpha_i^{(m)} - \alpha_i^{r}) e_i \|$

$\geq \ < \sum | \alpha_i^{m} - \alpha_i^{(r)} |$

$\sum_{i=1}^{n} | \alpha_i^{(m)} - \alpha_i^{r} | < \frac{\varepsilon}{c}$

$| \alpha_i^{(m)} - \alpha_i^{(r)} | \leq \sum_{i=1}^{n} | \quad | < \varepsilon / c$

$(\alpha_j^{(m)}) \Rightarrow | \alpha_i^{(m)} - \alpha_i^{(r)} | < \varepsilon / c = \varepsilon_1$

If $(\alpha_j^{m}) \to \alpha_j \quad (j = 1, 2 - n)$

let

$y = \alpha_1 e_1 + \cdots \alpha_n e_n \in Y$

$y_n \to y$ convergent and

Y is complete space.