normed

Space X

$$\|x\| = \sup_{\substack{f \in X' \\ f \neq 0}} \frac{|f(x)|}{\|f\|}$$

Th

2) If x_0 is $\ni$ $f(x_0) = 0$ for all $f \in X'$ then $x_0 = 0$

the

2) For every x_0 in a normed space X there is a bounded linear functional $\ni$ $\tilde{J}(x_0) = \|\tilde{J}\| \|x_0\|$ and $\|\tilde{J}\| = \|x_0\|$

Reflexive normed space ?

Isomorphise

$T : X \to Y$

1. linear $T(\alpha u + \beta v) = \alpha(T(u)) + \beta(T(v))$
2. Bijective
3. norm preserving $\|T(x)\| = \|x\|$

then X is IsomoR

(1) a normed space X

(2) dual space $X' = \{f \mid f : X \to K\}$. It is space of all bounded linear function on X

(3 Bidual $X'' = \{g \mid g : X' \to K\}$ It is the dual space $(X')'$ of X' and is also called Second dual space of X

In For a fixed $x \in X$ we defined a functional $g_x : X' \to K$

$$g_x(f) = f(x) \qquad f \in X'$$

we prove g_x is a bounded linear function on X' that g_x

cannonical maping

To every $x \in X$ there corresponds a unique bounded linear functional $g_x \in X''$ given by

$$C : X \to X'' \quad \text{as } C(x) = g$$

C is called canonical maping of X into X''

The ⇒ The cannonical maping $C : X \to R(C)$ is an isomorphism

Reflexive normed Space

A normed Space X is said to be reflexive if canonical maping

$C: X \to X''$ is Surjective

$R(C) = X''$

The $\Rightarrow$ of a normed space X is reflexive, It is a Banach (complete) space but converse does not hold

The $\Rightarrow$ every dual normed space is complete

$\Rightarrow$

$C[a,b]$ is complete but not reflexive

The $\Rightarrow$ every Reflexive normed space is complete.

therefore an incomplete normed space cannot be reflexive so

(1) Space of all polynomial with maximum space

(2) Space of all real continuous real valued function on $[a,b]$ with integral norm are not reflexive and not complete

The every finite dim normed space is reflexive

$\Rightarrow$ In case of a finite dim normed space, dim of a space and its dual space are space (same)

$\Rightarrow$ every finite dim normed space is reflexive like R, C, C^n, R^n

② Space of all nth degree polynomial zero polynomial etc are reflexive

The ⟹ every hilbert space H is reflexive

e.g R^n, l_2 are Hilbert space are reflexive

The ⟹ A separable normed space X with nonseparable dual space X can not be reflexive.

Lema

let Y be proper closed subspace of a normed space X. Let $x_0 \in X - Y$ be arbitrary and

$$\delta = \inf_{\bar{y} = Y} \|\bar{Y} - x_0\|$$

the distance from x_0 to Y then there exist an $\bar{f} \in X'$ ∋

$\|\bar{f}\| = 1$; $\bar{f}(Y) = 0$ $\quad Y \in Y$

$\bar{f}(x_0) = \delta$

The Separability

A separable normed space X with a nonseparable dual space X' cannot be reflexive