Nowhere dense

A subset M of a metric X is said to Nowhere dense in X if its $\bar{M}$ has no interior

$(\bar{M})^{o} = \varphi$ $\bar{M}$ contain no ball

e.g

(R, d) then $\bar{Z} = Z$, $(\bar{Z})^{o} = \varphi$

Hence nowhere dense Z in R

N and Q are also nowhere dense in R

$\bar{Q} = \overset{o}{Q}$

$(Q)^{o} = \varphi$

$\Rightarrow$ every singleton is nowhere dense $(\bar{a}) = a$

$(\bar{a})^{o} = \varphi$

{ Z is closed set

open interval is continuous but Z is discrete

First category

A subset M in a metric X is said to be first category union of countable many sets which all nowhere dense

e.g Q is countable and Q is countable union of nowhere dense is first category

M is of second catagory in X if is not of first catagory

The **Baire's catagory Theorem**

If a metric space $X \neq 0$ is complete It is second category Hence If $X \neq 0$ is complete

uniform boundeness Theorem

let $\{Tn\}$ be a ~~unique~~ seq of bounded L. operator $Tn: X \to Y$ from a Banach space X into a normed space Y. If for every $x \in X$, $\{Tn(x)\}$ is bounded

$\|Tn(x)\| \leq c_x \quad n = 1, 2 \ldots$ (point wise bounded

where c_x is real no, then seq $\|Tn\|$ is also bounded that is a positive real c ∃

$\|Tn\| \leq c \quad n = 1, 2 \ldots$ (uniform bound

v.v.v **Open maping**

let X and Y be metric space a maping $T: D(T) \to Y$ with domain $D(T) \subset X$ is called open maping If for every open set is $D(T)$ the image is an open set in Y

These are the maping ∋ the image of every open set is an open set

continous maping

A continuous mapping $T: X \to Y$ has the ~~same~~ property that for every open set in Y the inverse image is an open set in X

v.v

continu mapping are not neccesserly open maping

e.g

a maping $T: R \to R$ defined by $T(x) = \sin x$ A sine function continuous. T is continuous maping but $T[(0, 2\pi)] = [-1, 1]$ that is T maps an open set $(0, 2\pi)$ on to $[-1, 1]$ which is not open

eg

$T: R \to R$ by $T(x) = x^2$ then T is continuous map but not an open map because it map an open set $(-1, 1)$ onto $[0, 1)$ which is not open

⟹ A continuous linear operator need not be an open maping but what condition a conti linear operator is also an open maping

Ans open maping Theorem

The ⟹ open map theorem

A continuous (bounded) linear operator T from a Banach space X onto a Banach space Y is an open maping That is $T: X \to Y$ is a surjective bounded linear operator then T is open.

Bounded inverse theorem

Let X and Y be Banach space and $T: X \to Y$ bijective bounded linear operator. Then T^{-1} is continuous and hence bounded linear operator

Closed graph theorem

Intro

$X \times Y$ is a normed space

① $(x_1 . y_1) + (x_2 . y_2) = (x_1 + x_2, y_1 + y_2)$

② $a(x, y) = (\alpha x, \alpha y)$

$\|x, y\| = \|x\|_1 + \|y\|_2$ v.v. 9

Theorem

For any two banach space X and Y and X×Y is also a Banach Space

Closed linear operator

$T: D(T) \to Y$ be linear operator with $D(T) \subset X$. then T is called closed linear operator if its graph $G(T) = \{(x, y) : x \in D(T), y = T(x)\} \subset X \times Y$ is closed in the normed space X×Y

Theorem (Closed graph theorem)

Let X and Y be banach space and $T: D(T) \to Y$ a closed linear operator where $D(T) \subset X$ if $D(T)$ is closed in X then the operator T is bounded.

The Bounded inverse theorem

Let X and Y be banach spaces and $T: X \to Y$ bijective bounded linear operator.